Period 1, May 8, 2024

$$\int_0^{\frac{1}{2}} \frac{2}{\sqrt{1-x^2}}\,dx = 2\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\,dx$$

$$2\arcsin x + c \Big[_0^{\frac{1}{2}}$$

$\arcsin \frac{1}{2} = \theta$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}$

$\arcsin 0 = \theta$

$\sin\theta = 0$

$\sin 0 = 0$

$$2\cdot\frac{\pi}{6} - 2\cdot 0 = \frac{\pi}{3} - 0 = \frac{\pi}{3}$$

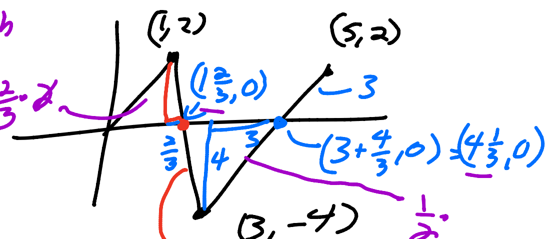
| | |
|---|---|
| y = Sin x | $[-\frac{\pi}{2}, \frac{\pi}{2}]$ |
| y = Cos x | $[0, \pi]$ |
| y = Tan x | $(-\frac{\pi}{2}, \frac{\pi}{2})$ |
| y = Sec x | $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$ |
| y = Csc x | $[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$ |
| y = Cot x | $(0, \pi)$ |

30, 27, 28, ~~8~~, ~~24~~, ~~18~~, ~~23~~, 26,

8. $\int_0^5 F(x)\,dx$

$\frac{1}{2}b\cdot h$

$\cancel{\frac{1}{2}}\cdot 1\frac{2}{3}\cdot\cancel{2}$




$3 = \frac{Rise}{Run} = \frac{4}{Run}$

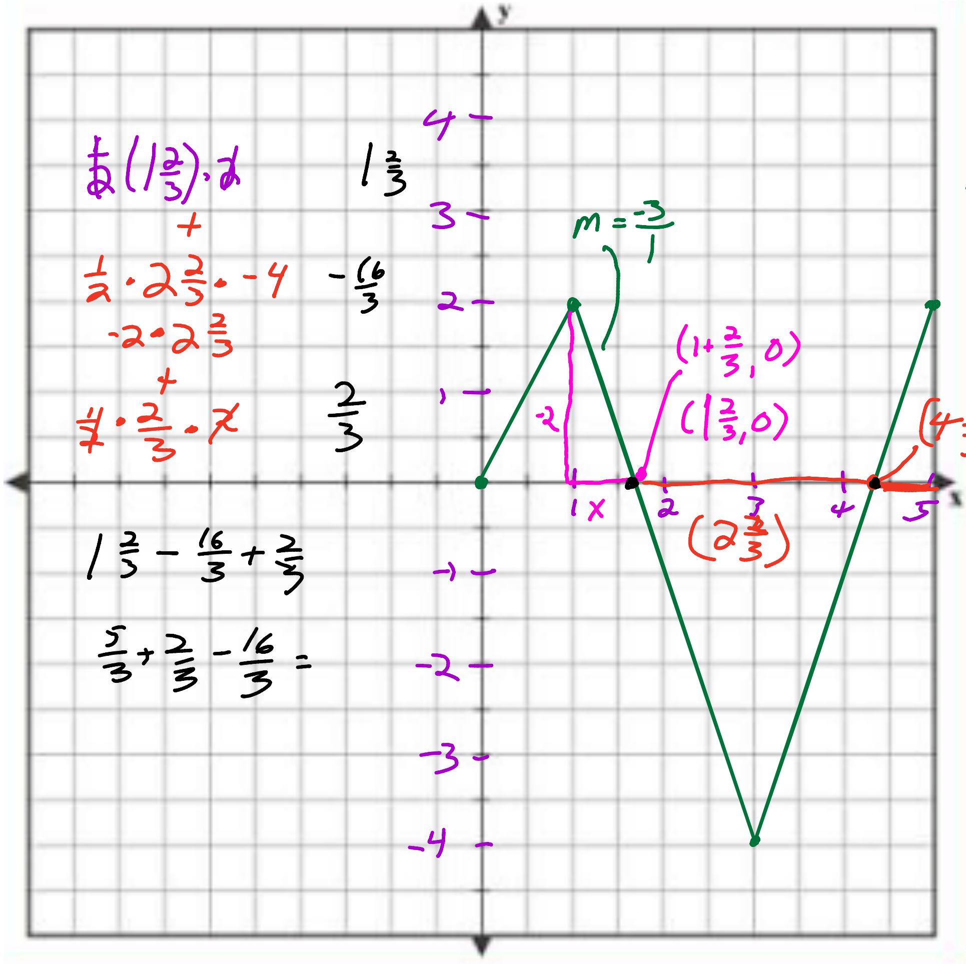
$Run = \frac{4}{3}$

$-3 = \frac{Rise}{Run} = \frac{-2}{Run}$

$Run = \frac{2}{3}$

$1\frac{2}{3}+$

$m = \frac{-6}{2} = -3$




$m=\frac{-6}{2}=-3$

$\frac{\text{Rise}}{\text{Run}}=\frac{-3}{1}$

$\frac{-3}{1}=\frac{-2}{x}$

$-2=-3x$

$\frac{2}{3}=x$

$4\frac{1}{3}-1\frac{2}{3}$

$\frac{13}{3}-\frac{5}{3}=$

$\frac{11}{3}=3\frac{2}{3}$

---

18. $V(T)=12T-3T^2$

Find when $V(T)=0$

$V(t)=3T(4-T)$

$0=3T(4-T)$ $\quad T=0,\ T=4$

$V(2)=12(2)-3(2)^2=+12$ $\quad V(6)=12(6)-3(6)^2=72-108=-$

$+ + + + + + + + +$ $\quad - - - -$

0 $\quad$ 2 $\quad$ 4 $\quad$ 6

Right $\quad$ Left

$\underset{\text{Right}}{32}=\int_0^4(12T-3T^2)\,dT=6T^2-T^3\Big|_0^4$

$6(4)^2-4^3=96-64=32-0$

$(6(6)^2-6^3)-(6(4)^2-4^3)=0-32=-32$ $\quad \int_4^6(12T-3T^2)\,dT=-32$ Left

23. $$\int \frac{\csc^2(\sqrt{x})}{\sqrt{x}}\,dx$$

$u = \sqrt{x} = x^{\frac{1}{2}}$

$du = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}\,dx$

$2\sqrt{x}\,du = dx$

$$\int \frac{\csc^2 u}{\cancel{u}} \cdot \cancel{2x}\,du$$

$$2\int \csc^2 u\,du = 2(-\cot u) + C$$

$$-2\cot\sqrt{x} + C$$

$8x^3 = (2x)^3$

$2x \cdot 2x \cdot 2x$

$8x^3$

24 $$\lim_{x\to 0} \frac{\tan^3 2x}{x^3}$$

$$\lim_{h\to 0} \frac{\sin h}{h} = 1$$

$8 = 2^3$

$$\lim_{x\to 0} \frac{\sin^3 2x}{\cos^3 2x} \cdot \frac{1}{x^3} = \lim_{x\to 0} \frac{8 \cdot \sin^3 2x}{8 \cdot x^3} \cdot \frac{1}{\cos^3 2x} = \lim_{x\to 0} \boxed{\frac{8\,\sin^3 2x}{(2x)^3}} \cdot \frac{1}{\cos^3 2x}$$

1

$$8 \cdot 1 \cdot \frac{1}{\cos 0} = 8 \cdot 1 \cdot 1$$

$8$

$$\lim_{x\to 0} \frac{\sin 2x}{2x} \cdot \frac{\sin 2x}{2x} \cdot \frac{\sin 2x}{2x} = \lim_{x\to 0} \frac{\sin^3 2x}{8x^3}$$

$1 = 1 \cdot \quad 1 \quad \cdot 1$

26

$$y = \left(\frac{x^3-2}{2x^5-1}\right)^4 \qquad \text{Find } \frac{dy}{dx} \text{ at } x=1$$

$$u = \frac{x^3-2}{2x^5-1}$$

$$\frac{du}{dx} = \frac{3x^2(2x^5-1) - (x^3-2)\cdot 10x^4}{(2x^5-1)^2}$$

$$y = u^4$$

$$\frac{dy}{du} = 4u^3$$

$$\frac{dx}{dx} = \cancel{\frac{du}{dx} \cdot \frac{dy}{du}}$$

$$\left(\frac{3x^2(2x^5-1) - (x^3-2)\cdot 10x^4}{(2x^5-1)^2}\right)\left(\cdot 4\left(\frac{x^3-2}{2x^5-1}\right)^3\right)$$

$$\frac{(3\cdot 1 - (-1)\cdot 10)}{1} \cdot 4\left(\frac{-1}{1}\right)^3$$

$$(3+10)\cdot 4(-1)$$

$$13 \cdot 4 \cdot -1 = -52$$

---

27.

$$\int x\sqrt{5-x}\,dx = \int (5-u)\cdot u^{\frac{1}{2}}(-du) = -\int \left(5u^{\frac{1}{2}} - u^{3/2}\right)du$$

$u = 5-x \quad \Rightarrow x = 5-u$

$du = -dx$

28. $\cancel{\frac{dT}{y}} \cdot \frac{dy}{\cancel{dT}} = -2\cancel{y}\,\frac{dT}{\cancel{y}}$ $\qquad y(0) = 100$

$$\int \frac{1}{y}\,dy = \int -2\,dT$$

$$\ln y = -2T + C$$

$$\ln 100 = -2(0) + C$$

$$\ln 100 = C$$

$$\ln y = -2T + \ln 100$$

$$e^{-2T + \ln 100} = y$$

$$e^{-2T} \cdot e^{\ln 100} = y$$

$$\frac{1}{e^{2T}} \cdot 100 = y$$

$$\frac{100}{e^{2T}} = y$$

$$\frac{100}{e^{4}} = y$$

---

30. $$\int_0^1 \tan x\,dx = -\ln|\cos x| + C$$

$$= \ln|\cos^{-1} x| + C$$

$$\ln\left|\frac{1}{\cos x}\right| + C$$

$$\ln(\sec x) + C$$

$\ln x^2 = 2\ln x$

$\cos 1 = \cos 1$

$\cos 0 = 1$